Force measurements on static granular materials

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A method for measuring normal forces under a granular packing is introduced. A high precision electronic balance is used to measure normal forces on individual beads at the bottom of a granular material. Due to the high sensitivity of this setup the normal forces on individual beads without applying any external load on the system were measured. From these measurements the distribution of forces, the response to small perturbations at grain level and the spatial correlation between normal forces were investigated at the bottom of the packing. The distribution function is consistent with a power law with an exponent $\alpha=0.3$ for small forces. At large forces a crossover to an exponential decay is observed with a decay constant $\beta=1.8$. Small amplitude spatial correlations were observed in the normal forces at the bottom of the pile, and the radial pressure distribution was dependent of the different filling procedures. Finally we report on systematic relaxations in the measured weight as a function of time on single grains at the bottom of the packing after perturbing the system. [S1063-651X(99)16411-0]

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I. INTRODUCTION

Granular materials are an important class of matter. In the static state they behave much like solids and in the flowing regime or liquid state they behave like fluids. But still granular materials are known to exhibit many unexpected properties [1] and are not regular liquids or solids.

Granular materials have been studied for more than two centuries [2]. In a study Reynolds, who investigated dilatancy in bead packs, found that a granular packing has to expand in order to undergo any shear deformation [3]. Surprising results were also found when the pressure inside a granular column was studied. The German engineer Janssen [4] found that in silos filled with grains the pressure becomes depth independent below a certain depth. This has also recently been studied by Vanel and co-workers [5]. Not only has the geometrical disorder in bead packs been studied [6,7] but also the closely related heterogeneity in the internal distribution of forces, where only a fraction of the beads in the packing bear a large proportion of the total load [8–10]. The force network has been investigated qualitatively in two (2D) and three dimensional compression cells where the stress paths or ''arches'' have been visualized by stress induced birefringence [8–10]. The directional ordering of stress inside granular materials is often referred to as the ''arching effect.''

While the above mentioned experiments have given insight in the internal geometrical structure of bead packs and force chains, it is at best difficult to get any quantitative information of the internal forces from these experiments. However, it is possible to obtain values of the forces exerted on the boundary of the packing. This has been done by means of the carbon paper method [11,12]. In these measurements the normal force at the boundary is measured by looking at the size of the spots the beads leave on a carbon paper. In the first experiments Liu *et al.* [11] found that the probability distribution function $P(F)$ for normal forces F decays exponentially for large forces. In an improved version of these measurements [12] the exponential decay is still present for forces larger than the mean force $\overline{F}$ in the system. And they find that the probability distribution flattens out and becomes more or less constant for $F<\overline{F}$. The coefficient in the exponential decay β is measured to be 1.5 ± 0.1. This group also investigated the spatial correlation between the forces at the bottom of the system which was found to be uncorrelated. Even if the carbon paper method is elegant, it is still rather limited since large external loads are needed to leave sufficient marks on the paper. It is therefore not suitable for measurements on systems which are just under influence of gravity like regular silo systems or sand piles.

The nature of the internal force network is of great importance in the understanding of various properties of granular assemblies. In order to explain some of the observed properties different theoretical models have been proposed, ranging from simple scalar models [13,14] to full tensorial descriptions [15].

A well known model is the q model first presented in [11] and discussed further in [14]. The q model is a scalar model which only takes the normal component of the force into account. This model has been studied on different underlying lattices. The force probability function $P(f)$ is analytically found to be of the form $P(f)\sim f^{N-1}e^{-Nf}$ [14]. Where $f=w/\langle w\rangle$, w is the weight and $\langle w\rangle$ is the mean weight at a given depth and N is the number of underlying neighbors in the lattice. This analytic form of $P(f)$ is valid in the limit of infinite depth and periodic boundary conditions.

Recent contact dynamic simulations [16–18] have been performed on both two and three dimensional systems. And again the probability distribution is found to be a power law for forces smaller than the mean force in the system and it has an exponential decay for larger forces. Furthermore the exponent β in the exponential part of the distribution is found to be 1.4 which is in agreement with the latest carbon paper measurements [12] and the results presented here. The distribution found in these simulations is also found to be quite robust with respect to changes in the dimensionality and grain-grain friction coefficient.

A main challenge in research on granular materials is to perform reliable force measurements. Force measurements are hard to perform because one has to move something to measure forces. And by moving the grains during the measurement one will also, to some extent, change the granular system at hand [19,20].

The aim of this work is to develop and apply a method to measure the forces on individual grains at the bottom of a granular system *without applying any external load*. With the setup we are reasonably able to control the perturbation induced by the measurements and to explore how the granular system responds to small perturbations at grain level. Measuring the radial pressure distribution we observed that a dip in the averaged grain forces under the center of the packing results when the grains are poured directly into the container but not when the container contains a sleeve which is subsequently removed. In contrast to existing measurements the present technique resolves small amplitude spatial force correlations among the grains at the bottom of the container. While the distribution of forces largely coincides with existing observations these correlations have not to our knowledge been previously observed. The present paper confirms the observed robustness of $P(f)$ by showing that it is also invariant under variations in filling procedure, and variations in probe position and the waiting time between individual measurements.

The paper is organized in the following manner. In Sec. II the experimental setup is described, in Sec. III the results are presented, in Sec. IV the results and experimental problems are discussed, and, finally (Sec. V), we sum up our main results and give some prospects of future investigations with this setup.

II. EXPERIMENTAL METHOD

The experimental system is sketched in Fig. 1. It contains a cylinder filled with glass spheres. The container is an open glass tube with inner diameter of 80 mm. The bottom of the tube is glued to a solid foot which rests on a table. The silo is then filled with 1.3 kg of 2 ± 0.05 mm glass beads which corresponds to $\sim$120 000 beads, or a filling height of $\sim$160 mm. In most of the experiments the system is filled by pouring the beads gently into the container. Afterwards we tap horizontally on the tube to relax the packing before starting the experiment. To check the influence on the measurements from the filling procedure we also filled the system in a different way. This was done by placing a smaller tube of internal diameter 60 mm and external diameter 65 mm inside the container. This tube was then first filled, and the container was filled by slowly removing the inner tube.

The silo or container is resting on a thick aluminum table as illustrated on Fig. 1. This table has a 1 mm hole in the center where a force probe is located. The probe is a flat pointed needle with a diameter slightly smaller than 1 mm. The probe is attached to our pressure sensor, a Mettler PM1200 electronic balance, which rests under the table. The resolution of the balance is 10^{-3} g in the range from 0 to 1200 g. This makes it possible to measure weights ranging from a few bead weights to nearly the whole system weight. The balance has a feedback mechanism that maintains the vertical position of the probe. It thus measures the force

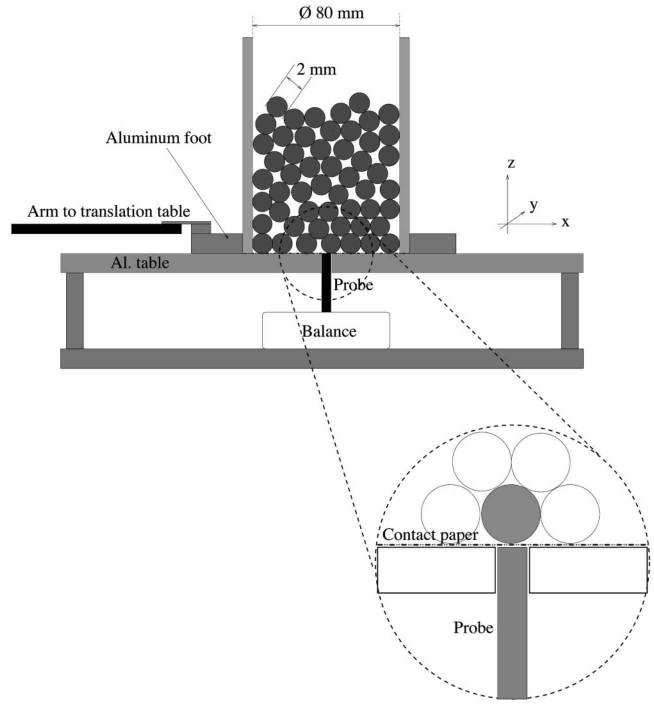

FIG. 1. Sketch of the experimental setup. The granular system and the container, the aluminum table, the probe, and the balance. One should also note the contact paper between the granular packing and the table. Forces at the bottom of this system are traced out by sliding the granular system over the table and probe.

needed to keep the probe in a fixed position. Since the movements of the probe are *small* it is well suited for pressure measurements. To be able to control the position of the probe tip it has a built in μm screw. In this manner the height of the probe tip could be adjusted to a precision of 1/100 mm. By changing the probe height we could change the perturbation on the granular material in a systematic way and explore its response to small perturbations at grain level. Since the measured weight is strongly depending on the position of the probe it is important to be able to control its position.

To shield the bottom layer of beads from shear with the underlying table a contact paper is placed between the beads and the underlying table. The bottom layer of beads is glued to this thin and flexible 3M Electro Cut contact paper, and the forces are measured through this plastic film.

With the probe tip near the table level we could trace out the normal forces acting on individual beads in the bottom layer by sliding the granular system over the table and the probe tip while measuring the pressure and the position.

The sliding of the system is done by connecting the foot of the container to a XY-translation table. This connection is a flat arm with steel blade springs in both ends. The arm is therefore stiff in the horizontal plane and flexible in the vertical direction. The arm is made this way to minimize the vertical forces transmitted to the granular system from the translation table. The translation table is controlled by a PC which simultaneously reads the balance. The normal forces from individual beads in an area are traced out by moving the system stepwise on a regular lattice where the normal force is measured in each grid point. In this manner we measure the forces on and obtain the position of the individual beads at the bottom of the packing. We typically move the

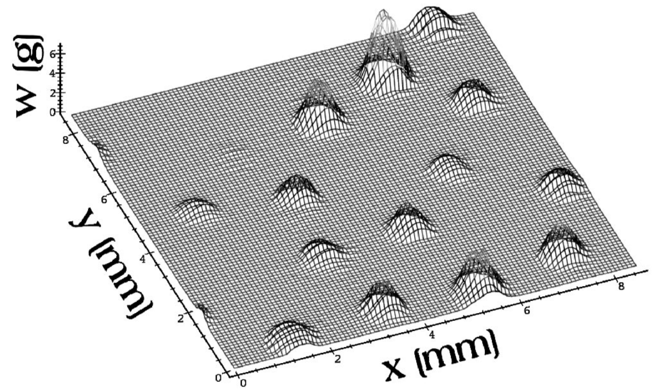

FIG. 2. 3D plot of data obtained by scanning a region under the packing. Position in mm along the X and Y axis and measured weight on the Z axis $[g]$. The individual beads are clearly visible as peaks in the landscape. One may also see that the measured forces on the peaks are widely distributed. The Gaussian shape of the measured peaks is due to the fact that the point of the probe is slightly rounded.

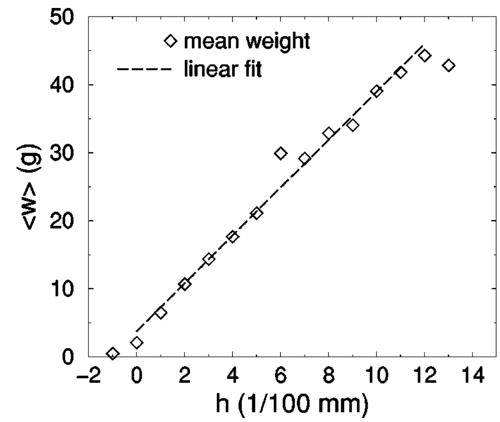

FIG. 3. Graph of the response to different probe positions. The mean weight $\langle w\rangle$ in an area is plotted against the probe height above table level h.

system ~ 0.15 mm between each measurement.

The data are collected in matrixes with the measured weight and the position of each measurement. Data from a small scan are plotted in Fig. 2. The smoothness of the particle peaks in this plot shows that the reproducibility in the setup is good, since each scan takes several hours to perform. When the reproducibility of these measurements was studied it turned out that the position reproducibility was very good on all time scales. The amplitude reproducibility was good within hours but after days the amplitude on individual grains could change. The observed robustness of the distribution function implies that the system is unchanged in a statistical sense. The experiments are performed at room temperature $T=22°\pm 2°$, under dry conditions with relative air humidity less than 35%. It should be noted that the temperature and humidity are not strictly controlled as some authors point out could be necessary [19,5]. However, our system is not so sensitive to temperature variations since the top surface is free and the whole system is ''floating'' on top of the table, and the packing is free to expand in the vertical direction. To check this we compared experiments performed at different temperatures. No significant changes were observed in the force distribution function.

III. RESULTS

A. Changes in probe position

In order to investigate the effect of probe height variations on the measurements we measured the mean weight $\langle w\rangle$ for different probe positions. This was done with repeated measurements with different probe heights while the system was otherwise unchanged.

In Fig. 3 the mean weights are plotted against the height of the probe tip above the table level. For probe heights in the range from 0 to 0.12 mm a linear dependency on the mean weight in the system with the probe position was found. This implies an elastic response in the mean normal force, within a limited range, to perturbations at grain level.

We then looked for changes in the force distribution function for different probe heights. To check this, similar measurements were performed on many large systems with different probe heights. The measured weights w in each dataset were scaled with the mean weight $\langle w\rangle$ in the system and the force distribution function $f\equiv w/\langle w\rangle$ was calculated, in the same way as in [12]. In Fig. 4 the calculated distribution functions $P(f)$ are plotted on a log-log scale for four different probe heights. From this we conclude that the force distribution function is unchanged under these perturbations when the forces are scaled with the mean force in the system.

B. Distribution of forces

By using data containing $\sim 30\,000$ measured peak weights the force probability distribution function was calculated. The obtained distribution function $P(f)$ is plotted on a lin-log scale in Fig. 5 and on a log-log scale in Fig. 6.

The obtained distribution function $P(f)$ is consistent with the form

$$P(f)\propto\begin{cases} f^{\alpha} & \text{if } f<1\\ e^{(-\beta f)} & \text{if } f>1.\end{cases} \tag{1}$$

From linear regression of log transformed data the coeffi-

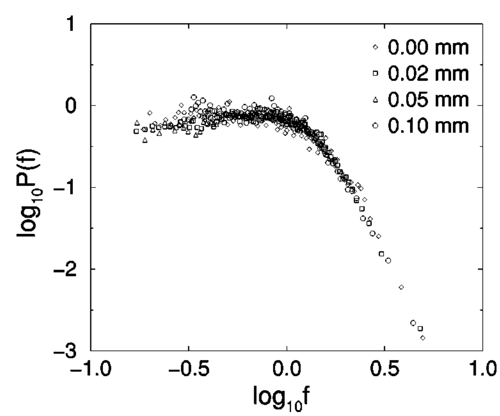

FIG. 4. Log-log plot of probability distributions $P(f)$ obtained with different probe heights, $f=w/\langle w\rangle$ is the scaled weight. As one could see they fall more or less on top of each other when scaled with the mean weight in the system. There are deviations in the measured distribution functions but the changes are *not* systematic with probe position.

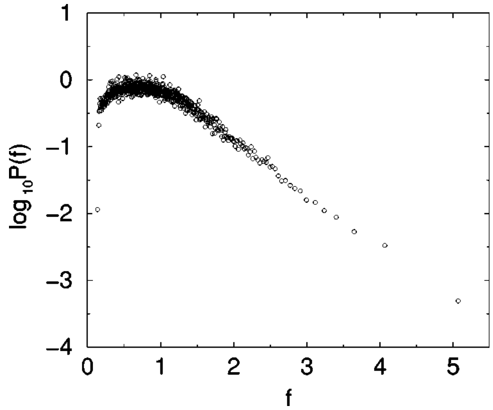

FIG. 5. Lin-log plot of the calculated probability density function.

cients α and β were found to be 0.3 ± 0.2 and 1.8 ± 0.2, respectively. The value of β is in reasonable agreement with the exponent found in similar experiments [12], and 2D and 3D contact-dynamic computer simulations [16,17]. By assuming a power-law distribution for $f<1$ we are able to give an estimate for the exponent in the "power law." A power-law behavior was also reported in the simulations by Radjai and co-workers [16,17], but the calculated exponent in this case was negative.

C. Spatial correlation

To investigate the spatial force correlations, the radial force-force correlation function $G(r)$ defined in the same manner as in [12], was calculated.

$$G(r)\equiv\frac{\sum_{i=1}^{N}\sum_{j>i}\delta(r_{ij}-r)f_if_j}{\sum_{i=1}^{N}\sum_{j>i}\delta(r_{ij}-r)}, \tag{2}$$

where r_{ij} is the distance between bead i and j, and $\delta(0)=1$. The normalized normal force acting on bead i is f_i, and N is the total number of beads in the data set.

The calculated force-force correlation function is plotted in Fig. 7 together with the corresponding number $N(r)$ of

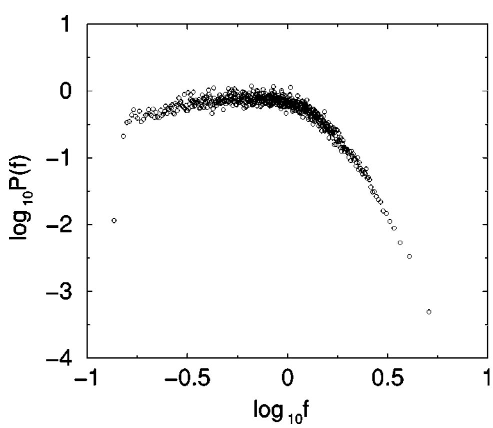

FIG. 6. Log-log plot of the calculated probability density function.

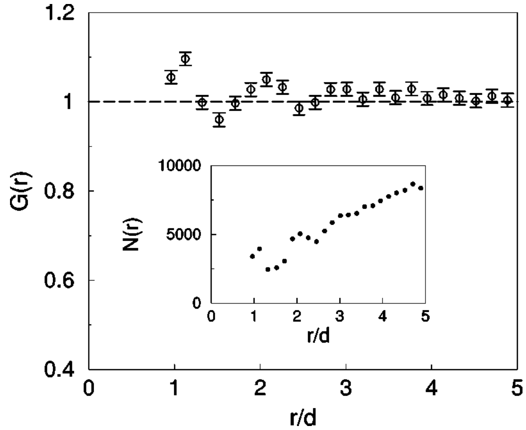

FIG. 7. On the main graph the calculated force-force correlation function $G(r)$ is plotted. The pair separation r is given in terms of bead diameters d. The error bars are estimated by looking at variations in the calculated correlation function for different data sets. In the inset the number $N(r)$ of force pairs (f_i,f_j) corresponding to the calculated correlation function is plotted.

force pairs (f_i,f_j) as function of distance. A "force pair" is two normal forces f_i and f_j separated by r_{ij} which together contributes to the force-force correlation function.

Small oscillations in the the correlation function is observed which decrease with increasing r/d. Even if these oscillations are small, in the order of 5%, this indicates weak correlations between the forces.

D. Radial force distribution

The radial pressure distribution was investigated for the two different filling procedures. First for the system where the grains were gently poured into the container, and then for the system where we filled an inner tube which was then removed. It is interesting to note that we were able to see a difference between these filling procedures.

As the plotted data show (Fig. 8), the radial pressure distribution is depending on the packing history. In the case where we pour the beads in directly, there seems to be a

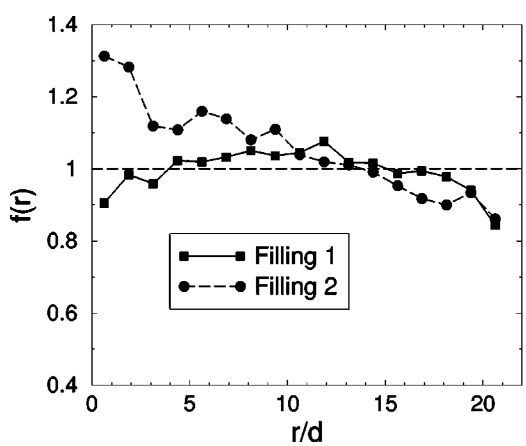

FIG. 8. The radial pressure distribution for the two filling procedures. For the initial system where the beads are poured gently into the container (1) and the system where an inner tube is filled and then gently removed (2). The radial distance is plotted in units of bead diameters d. At the center of the tube $r=0$.

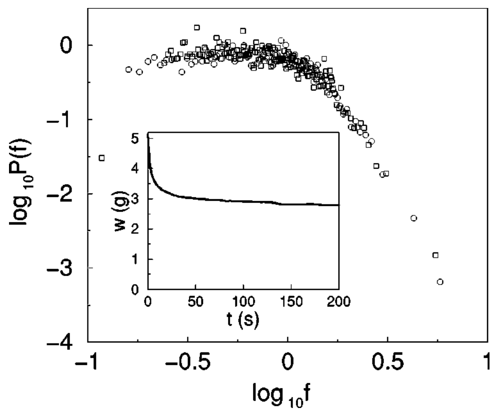

FIG. 9. On the main graph the force probability density for 2 and 4 sec waiting time are plotted on the log-log scale (circles and squares respectively). In the inset, data from a measurement on the relaxation process after perturbing the system are plotted. Here the measured weight when a single bead is resting on the probe tip are plotted as function of time after the perturbation. The system is perturbed by moving the bead to the probe before starting the measurements.

pressure dip in the center, a more or less constant level for larger distances and finally the pressure drops near the container walls. In the later case where we first fill the inner tube and then slowly remove it, we have the largest pressure in the center and the pressure falls off monotonously towards the walls. Even if we observe changes in the radial pressure distribution we are not able to see any changes in the earlier presented force distribution function $P(f)$, which seems to be unaffected by these changes. The oscillations in the force-force correlation function is also unaffected by the changes in packing history.

E. Relaxation process

While studying the reproducibility in the setup we observed a relaxation in the system after perturbing it. The measured weight would decrease with time towards a constant value after moving the system so that the sensor was under a bead. Typical data for the time evolution of the measured weight w are plotted in the inset in Fig. 9. From the plot we could see that the measured weight relaxes significantly during the first minute after the movement of the system. Thereafter the measured weight is more or less constant. But even if it seems to be a consistent relaxation process from bead to bead it is hard to fit the data with an analytical function.

Because of this relaxation in the pressure after moving the system it is conceivable that the time we wait between each movement of the system would change our results. To check this we doubled the waiting time and performed similar experiments. These data were then compared to our initial data. The obtained distribution function for four seconds waiting time is plotted together with data collected with two seconds waiting time in Fig. 9. From this test we concluded that the distribution of forces was unchanged when we doubled the waiting time. If we look at Fig. 2 we could also see that the peaks in the data set are quite smooth even if the time between measurements on the same bead could be long. This is because we perturb the system in the same manner and wait the same amount of time between all measurements.

IV. DISCUSSION

An important result from our measurements is the force distribution function for forces smaller than the mean force. In the case $f<1$ the distribution is consistent with $P(f) \propto f^{\alpha}$ with $\alpha>0$. This lower part of the distribution function is different from what has been measured by other experiments [12] and results from numerical simulations. However, the exponential decay in the distribution is consistent with what has been observed experimentally [12] and found in numerical models [16,17,11].

When comparing distribution functions obtained from recording *all* grain–grain forces in numerical simulations to experimental results, one should always keep the geometrical differences in mind. In our experiment only the forces at the the bottom layer of beads are recorded. These beads are forced to be in the same plane and this makes the geometry at the boundary different from the geometry in the bulk. At the bottom layer all the contact normals are pointing directly upwards whereas beads in the bulk have several underlying particles [21].

It is instructive to try and connect the bulk and wall results: Roughly speaking, one wall contact is equivalent to $z/2$ bulk contacts, where z is the mean coordination number. For small forces and $z=4$ this gives the following relation between the bulk normal force distribution $p(v)$ and our measured distribution $P(f)$:

$$P(f)=\int_0^f p(v_1)p(v_2)\,\delta(f-v_1-v_2)\,dv_1\,dv_2, \tag{3}$$

where v_1 and v_2 are the normal forces from the two underlying beads which is replaced by the single wall contact. By assuming power laws for $P(f)$ and $p(v)$ with exponents α, and α', respectively, the exponents are related through

$$\alpha=2\alpha'+1. \tag{4}$$

The two dimensional simulation result presented in [16] gives $\alpha'=-0.3$, which corresponds to $\alpha=0.4$. This is consistent with our result. Note, however, that these simulations have been performed in two dimensions while our experiments are three dimensional.

The main advantage of this method is its sensitivity to small forces, which allows the measurements of forces that arise intrinsically in the granular systems *without an external load.* It is also important that we are able to control and change the perturbation induced on the system by changing the height of the probe tip. With this technique we are able to explore the system's response to perturbations at grain level and to see how it changes the statistical quantities.

It is remarkable that the obtained distribution function is rather similar to what has previously been found with carbon-paper measurements, since these measurements were done under very different conditions from our experiments. In the carbon paper measurements a high pressure is exerted on top of the granular packing, while there is no external load on our system which is just under gravity. The smallest forces measured in the carbon-paper experiment are of the same order as the largest ones found in our setup. It is not obvious that the distribution function is unaffected by the boundary conditions and system preparation.

Furthermore the measured force distribution function seems to be very robust. It does not change significantly or systematically if we change the position of the probe tip or use a different filling procedure. Even a changed waiting time between each measurement does not affect the distribution function.

One could, of course, discuss how much the system changes during the measurements since a single scan takes several days. It is conceivable that individual normal forces could change during the measurement. But since the data plotted in Fig. 2 take several hours of measuring we argue that these changes have to take place on time scales in the order of days. And from the robustness of the force probability distribution function this indicates that these slow changes are random and tend to be averaged out. We therefore expect that the statistical quantities are unchanged during a scan, and that our data could be viewed as a ‘‘time average’’ of the system. A different average is taken in the carbon-paper measurements [12]. In this case it is an integrated average through the ‘‘pressing process’’ where the largest force in each point during the ‘‘pressing process’’ is measured. So neither our method nor the carbon paper method gives a snapshot of the normal forces at the boundary. Our observation of the robustness of the force probability distribution function could imply that the force probability function is not very sensitive to the boundary conditions.

Regarding the calculated correlation function, the observed ripples in is of the order of 5% and is reproducible. The oscillations are also robust with respect to changes in filling procedure.

Probably grains of certain spacings are more likely than others to share a common force arch that distribute the above forces evenly among them. A further insight into these correlations represent a challenge to simulations as well as further experiments.

We have further shown that the radial pressure distribution in our system depends on the packing history. This indicates that the developed experimental method is well suited to study pressure distributions under various granular systems under influence of gravity, to explore the dependency on packing history and ‘‘pile geometry.’’

The observed relaxation in the packing after perturbing the system is an interesting effect, and it must be an effect intrinsic to the granular medium as it was absent in all our calibration measurements. Further experiments are needed to understand the physics behind this behavior.

V. CONCLUSION

We have developed a method for measuring normal forces at the bottom of a granular system. With this setup we are able to measure forces without applying external loads on the system. By changing the height of the probe tip we are, to some extent, able to control the perturbations on the system. The main limitation in the developed method is the time needed to collect the data.

We have also been able to measure the spatial correlation function $G(r)$ and found that there are small amplitude correlations in the normal force on the beads at the bottom of the system.

We found the response in the mean normal force to be linear in a small vertical displacement of the grains. Furthermore, the distribution of forces is unchanged under this perturbation when the individual forces are scaled with the mean normal force $\langle w \rangle$ in the system. The calculated probability density function could be fitted with a power law for forces smaller than the mean force in the system and decays exponentially for forces larger than the mean force in the system. The corresponding exponent α and the coefficient β are found to be 0.3 ± 0.2 and 1.8 ± 0.2, respectively.

In the future this setup will be used to study the pressure distributions under various granular systems. The observed relaxation in the packing after perturbing the system is interesting and should be investigated in further studies.

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